Research News — Meson scattering at high precision

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Abstract

A fascinating new generation of experiments has determined certain meson scattering parameters at high precision. A confluence of highly sophisticated theory as well as new experimental ideas have led to this state of affairs, which sheds important light on the properties of the strong interactions. A brief review of the experiments and the theory is presented.

The NA48 collaboration in CERN has recently provided information through two independent measurements of combinations of so-called scattering lengths associated with the scattering of two pions. The experiment is based on in CERN's highest-intensity proton beamline and uses a large and sophisticated detector.

The first measurement by the NA48/2 experiment [1] comes from a recently proposed idea due to Cabibbo [2] to measure a 'cusp' in the invariant mass distribution of pions resulting from the decay of the kaons. A cusp at an energy corresponding to $2m_{\pi^+}$ in the number distribution of the neutral pion pair as a function of their invariant mass, manifests itself as an abrupt change in the derivative of the number distribution. It is a very fine effect and can be seen only if the sample size of events that are analyzed is very large. The event sample here is enormous and is based on about 27 million 'events' $K^\pm \to \pi^\pm\pi^0\pi^0$. In units in which the mass of the charged pion m_{π^+} is set to unity, they obtain for the combination of scattering lengths $|a_0^0 - a_0^2|$ the value 0.264 ± 0.015, an accurate measurement of what was a rather poorly measured experimental quantity.

In the expression above, the a_0^I, $I = 0, 2$ are called scattering lengths, the subscript denoted by I stands for 'iso-spin' and the subscript 0 denotes the fact that this is the scattering length of the angular momentum $l = 0$ channel (S-wave scattering length). Pions, which come in three varieties π^+, π^-, and π^0, are the lightest of all strongly interacting particles, and are bound states of quark and anti-quark pairs of u- and d- varieties. Heavier counterparts where one of these is replaced by the s- quark are the kaons. Pions hold the key to our understanding of the strong interactions, which are resisted analytic solution in the low-energy domain. The strong interactions at the microscopic level are described by quantum chromodynamics (QCD) which is a theory in which the degrees of freedom are the quarks and gluons, while at macroscopic length scales one observes mesons (pions, kaons, etc.) and baryons (protons, neutrons, hyperons, etc.). The lightness of the pions on the hadronic scale ($m_{\pi^0} \simeq 135$ MeV, $m_{\pi^+} \simeq 139$ MeV (we have set the velocity of light, $c = 1$, a common convention)) is today understood in terms of a phenomenon called spontaneous symmetry breaking of a global symmetry. It is common in scattering experiments to decompose the scattering amplitude, *viz.*, namely the mathematical expression that governs the strength with with a projectile is scattered into a specific direction by a target, into 'partial wave amplitudes' each of which has a definite angular momentum $l\hbar$; the S- wave corresponds to the part that is independent of the scattering angle. Finally, we note here that the pions lie in an 'iso-triplet', correspond to isospin 1. Therefore pion-pion scattering amplitudes could carry isospin of 0, 1, 2 (addition of two isospins I_1 and I_2, imply that the total isospin could lie between I_1+I_2 and $|I_1-I_2|$). The advantage of the iso-spin amplitudes is that the amplitudes for all the physical processes, $\pi^+\pi^+ \to \pi^+\pi^+$, $\pi^-\pi^+ \to \pi^-\pi^+$, $\pi^-\pi^- \to \pi^-\pi^-$, $\pi^+\pi^- \leftrightarrow \pi^0\pi^0$, and $\pi^0\pi^0 \to \pi^0\pi^0$ may all be expressed in terms of these, when the mass difference of the charged and neutral pions is neglected.

The second technique employed by the NA48 collaboration, results from which are still preliminary, comes from a rare decay of kaons in which a lepton and anti-lepton pair and two pions are produced in the decay. The rescattering of the pions in the final state can actually be observed and provides a sensitive laboratory for the strength of the interaction of these particles. The names of the famous scientists A. Pais and S. Treiman, and those of N. Cabibbo and A. Maksymowicz are associated with this effect. Based on this technique and on the analysis of 370, 000 decays a prelimary number for the scattering length a_0^0 is given as 0.256 ± 0.011, according to summary talks posted on the web-site of the collaboration in September 2006. The E865 experiment at the Brookhaven National Laboratory, USA also uses the rare kaon decay to analyze 400, 000 events and measured this quantity to be 0.216 ± 0.015 [3]. These two experiments in addition to low-energy phase shift also rely on what is known as Roy equation analysis which is described in some detail later in this article.

Another important experiment called DIRAC which stands for the Di-Meson Relativistic Atom complex is an experiment that uses highly sophisticated experimental techniques to get two charged pions to bind through the electromagnetic interaction to form a so-called pionium atom which then scatters into two neutral pions in the ground state of the atom, upon which the electromagnetic interaction is switched off and the neutral pions then scatter off. The lifetime of this state provides an accurate measurement of the same difference of two scattering lengths as in the cusp experiment of NA48, an effect predicted in a different setting over 50 years ago by a highly distinguished set of authors: S. Deser, M. L. Goldberger, K. Baumann and W. E. Thirring [4]. Based on a harvest of 6, 600 pionium atoms the experiment reports the value of $0.264^{+0.033}_{-0.020}$ [5], from a measurement of the lifetime of the ground state of ~ 2 fs. More data from the experiment is presently being analyzed to bring down the uncertainties.

Pion-pion scattering has long occupied the attention of theorists even before the advent of QCD. The reason for this was that it provided a paradise for theoreticians due to the simplicity of the process, and the possibility of deploying many powerful theoretical constraints that follow from general principles. Notable amongst these was the application of dispersion relations, relations which follow from the principle of causality in field theory. Loosely speaking dispersion relations arise from the application of Cauchy's theory of complex variable theory to scattering amplitudes, when the latter are considered as complex functions of complex energy arguments. Other principles go under the names of 'crossing symmetry' and unitarity. In the context of pion-pion scattering a system of dispersion relations were established that entailed the presence of certain unknown functions of the momentum transfer which limited the power of the dispersion relations. In an extraordinary feat in 1971, S. M. Roy used all the general properties of scattering amplitudes to eliminate all these problems, and gave a representation that required the knowledge of the two scattering lengths only, in addition to the knowledge only of the imaginary parts of the partial waves [6]. This further led to a system of coupled integral equations for all the partial waves of pion-pion scattering. However, partial knowledge of the low-lying waves and some theoretical models of the higher waves could be used to produce a determination of pion scattering lengths. This program to pin down pion scattering phase shifts came to be known as Roy equation analysis. The analysis of phase shift information provided by the rare K_{l4} decay and Roy equation analysis was used to pin down a_0^0 to the range 0.26 ± 0.05 based on 30, 000 events from the Geneval-Saclay experiment [7], when activity in the field stopped for a couple of decades.

After the advent of QCD and the subsequent development of low energy effective theories for pion-pion scattering there was a resurgence of interest in the subject. These effective theories exploit the symmetry properties of the strong interactions to provide a consistent framework as an expansion in powers of momenta and the quark masses has come to be known as chiral perturbation theory, and is identified with the work of J. Gasser and H. Leutwyler [8]. At leading order in the low-energy expansion, S. Weinberg gave a prediction for a_0^0 of 0.16 [9], while at next to leading order the number was revised to 0.20 ± 0.01, which resulted from the comprehensive analysis by Gasser and Leutwyler. The presence of light particles in the spectrum was the culprit for this substatial revision. The revision was found to stay stable at next to next to leading order, for a thorough discussion, see, e.g. ref [10]. Debates were sparked on what exactly was nature of the QCD ground state which would protect this prediction; deviations from conventionally accepted picture of spontaneous symmetry breaking due to which pions themselves arise as near massless states were proposed. Such scenarios would have predicted higher values of a_0^0 [11]. We note here that a comprehensive Roy equation analysis tailored to meet the needs of modern effective field theories was recently presented, see ref. [12].

These dispersion relations are sufficiently general to permit an extension into the complex energy plane. Typically, the existence of singularities known as poles on the second Riemann sheet represents the formation of bound states of quarks and anti-quarks, and imply the formation of an intermediate unstable particle. From the real and imaginary parts of the pole position one may deduce the mass of this new particle and its 'width' which is related to the inverse lifetime of the particle. Recently, using the properties above, along with the accurately known solutions of the Roy equations the pole position of the state known as σ has been determined to high accuracy in the pion-pion $I = 0$ channel. Note that this is a model independent way of establishing of what is the lowest-lying state in the strong interaction spectrum [13]. The mass and width are respectively given as $M_\sigma = 441^{+16}_{-6}$ MeV and $\Gamma_\sigma = 544^{+25}_{-18}$ MeV. Special attention may be paid to the small uncertainties.

An extension of the study to a more complicated process that involves the scattering of pions and kaons has also been carried but due to paucity of experimental data is yet to see the spectacular success of pion-pion scattering. Here the comparison of dispersion relations and chiral perturbation theory has been performed in ref. [14]. A modern Roy-Steiner analysis, the analog of pion-pion scattering Roy equation analysis was recently provided in ref. [15]. On the experimental side, there is a proposal to produce πK atoms, while there is possibility of measurement of certain phase shift information at the COMPASS experiment. Much work remains to be done. However, the position of the κ-resonance has also been recently determined [16], using the principles analogous to those that were described in the pion-pion case earlier, using the accurately known solutions to the Roy-Steiner system.

There are studies on the lattice of scattering lengths and these have been reviewed in ref. [17] and is beyond the scope of the present article.

To summarize, we have pointed out the results emanating from a series of beautiful new experiments, and shown the confluence of theory and experiment. The theory uses effective field theories, dispersion relations, and together they make possible precise predictions in a domain long considered too inhospitable for such a state of affairs.

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